1 ENERGY STORAGE

1.1 A water well is in a location with a mean daily solar radiation of 4.8 hours of sun. The PV powered pump is able to produce 16m^3/day from a depth of 18m.

a) Determine the required capacity of the batteries for autonomy of 3 days.

An expression for gravitational potential energy:

$$E_{potential} = mgh$$

And in our case:

$$E_{pump,day} = m_{water} g h_{depth}$$

The parameters are:

$$m_{water} = \rho_{water} V$$
$$\rho_{water} = 1000 \frac{kg}{m^3}$$
$$V_{day} = 16 \frac{m^3}{day}$$

The mass of the water pumped daily is:

$$m_{water,day} = \rho_{water} V_{day} = 1000 \frac{kg}{m^3} \times 16 \frac{m^3}{day}$$
$$= 16000 \frac{kg}{day}$$

And as such the energy associated to this is:

$$E_{pump,day} = m_{water} g h_{depth}$$
$$g = 9.8 \frac{m}{s^2}$$
$$h_{depth} = 16m$$

$$= 16000 \frac{kg}{day} \times 9.8 \frac{m}{s^2} \times 16m$$
$$= 2.51 \times 10^6 \frac{kgm^2}{s^2} \frac{1}{day} = 2.51 \times 10^6 \frac{J}{day} = 0.7 \frac{kWh}{day}$$

Thus the pumping energy for three days (i.e. the autonomy for when there is no sun) is:

$$E_{pump,day} = 0.7\frac{kWh}{day}$$

$$E_{battery} = E_{pump,day} \times d_{autonomy}$$
$$d_{autonomy} = 0.7\frac{kWh}{day} \times 3day$$
$$= 2.1kWh$$

Ideally the batteries would have to have a total capacity of at least $2.1kWh$. However this is not taking into account losses that occur when a battery is charged and discharged.

b) Determine the required installed PV power.

The mean daily solar radiation of 4.8 hours of sun, i.e. the total energy received daily per unit area is:

$$t_{sol} = 4.8\frac{h}{day}$$
$$E_{sol,day} = 1000\frac{W}{m^2}t_{sol}$$
$$= 4800\frac{Wh}{m^2 day} = 4.8\frac{kWh}{m^2 day}$$

We want to know how much PV power has to be installed to be able to pump the required amount of water daily. As such the PV power has to produce the energy required to pump the water:

$$E_{PV,day} = E_{pump,day}$$

The energy produced per unit of installed power is given by:

$$E_{PV,day} = W_p \times t_{sol}$$

Knowing how much we have to produce and how many hours of solar radiation we can work out the power:

$$W_p = \frac{E_{PV,day}}{t_{sol}}$$

$$t_{sol} = 4.8\frac{h}{day}$$
$$E_{PV,day} = E_{pump,day} = 0.7\frac{kWh}{day}$$

$$W_p = \frac{0.7\frac{kWh}{day}}{4.8\frac{h}{day}} = 0.15kW$$

That is, 0.15kW of power in an area with 4.8h of mean daily solar radiation will produce 0.7kWh in one day.

1.2 How much water would have to be pumped to a tank raised 3 meters from the ground in order to be able to recover 1kWh of electricity? [Assume 100% conversion efficiency.]

What do we know:

$$h = 3m$$
$$E = 1kWh = 3.6MJ$$

Start with the expression describing Gravitational Potential Energy:

$$E_{potential} = Mgh$$
$$M = \frac{E_{potential}}{gh}$$

$$E_{potential} = E = 3.6MJ$$

We need to find the mass of water that has to be elevated:

$$M = \frac{3.6\times10^{6}J}{9.8\frac{m}{s^2}\times3m} = \frac{3.6\times10^{6}kg\frac{m^2}{s^2}}{9.8\frac{m}{s^2}\times3m} = 1.22\times10^{5}kg$$

And because the question asks for volume of water:

$$\rho = \frac{M}{V}$$
$$\rho_{water} = 1000\frac{kg}{m^3}$$
$$\therefore V = \frac{1.22\times10^{5}kg}{1000\frac{kg}{m^3}} = 122m^3$$

*For those curious about units. Energy can be defined the product of distance and force.
$E = Fd$
The force due to gravity is the product of mass and acceleration.

$$F = mg$$

And therefore:

$$E = Fd$$
$$F = mg$$
$$\therefore E = mgd$$

And the units are:

$$[E]=[mgd]=kg\frac{m}{s^2}m=kg\frac{m^2}{s^2}$$

2 MODULE TEMPERATURE

The electrical power is a product of current and voltage. Because the current and voltage of a Si solar cell are temperature dependent, the power output will vary with temperature.
For a Si solar cell, the open circuit voltage decreases with increased temperature and typically takes the following value:

$$\frac{dV_{oc}}{dT_c}\approx -2.3\frac{mV}{°C}$$

Assuming a typical open circuit voltage, the relative variation is:

$$V_{oc}=550mV$$

$$\frac{1}{V_{oc}}\frac{dV_{oc}}{dT_c}\approx\frac{-2.3\frac{mV}{°C}}{V_{oc}}=-0.004\frac{1}{°C}$$

A simple explanation for this decrease is that with increased temperature, the intrinsic free carrier density increases because are more e^-/h^+ pairs being generated by a) thermal excitation (k_BT) and b) the bandgap decreases with temperature facilitating thermal e^-/h^+ generation.
The increase in intrinsic carrier density is significant because the effect is to increase the saturation current:

$$I_s \propto n_i^2$$

The effect is to in turn decrease in the open circuit voltage according to the relationship we have already used:

$$V_{oc}=\frac{k_BT_c}{q}\ln\frac{I_L}{I_s}$$

Thus (ignoring other effects) if the saturation current increases, the open circuit voltage decreases.
Given that bandgap decreases with temperature, one would expect the short circuit current to increase because of an increase in absorption of light. However, this effect is comparatively small:

$$\frac{1}{I_{sc}}\frac{dI_{sc}}{dT_c}\approx 0.0006\frac{1}{°C}$$

Now because we can also describe the power output of a solar cell by the product of the open circuit voltage, short circuit current and fill factor, we can also characterise the temperature dependence of the fill factor (notice that it too decreases, like the open circuit voltage):

$$\frac{1}{FF}\frac{dFF}{dT_c} \approx -0.0015\frac{1}{°C}$$

Given that the maximum power can be described by the following product:

$$P_{\mathrm{m}} = I_{sc}V_{oc}\,ff$$

Then the variation in the maximum power is:

$$\frac{P_{\mathrm{m,var}}}{P_M} = \frac{1}{P_M}\frac{dP_M}{dT} = \frac{1}{V_{oc}}\frac{dV_{oc}}{dT_c} + \frac{1}{I_{sc}}\frac{dI_{sc}}{dT_c} + \frac{1}{FF}\frac{dI_{sc}}{dT_c}$$

If we put in the numbers shown above we obtain:

$$\frac{P_{\mathrm{m,var}}}{P_M} = \frac{1}{P_M}\frac{dP_M}{dT_c} = -0.004 + 0.0006 - 0.0015$$
$$= -0.005\frac{1}{°C}$$

The absolute variation of the power is therefore:

$$dP_M\left(T_c\right) = P_M P_{\mathrm{m,var}} dT_c$$

And the therefore the power as a function of temperature is

$$P_M\left(T_c\right) = P_M + P_M P_{\mathrm{m,var}} dT_c$$

You will notice that the greatest contributor to the temperature dependence of power output is the open circuit voltage. Therefore, for the sake of simplicity, we can assume that it is the only contributor:

$$P_M\left(T_c\right) = P_M + P_M\frac{dV_{oc}}{dT_c}dT_c$$

and that the maximum power point voltage V_m scales in the same way:

$$\frac{dV_m}{dT_c} = \frac{dV_{oc}}{dT_c} = -2.3\frac{mV}{°C}$$

$$\therefore \Delta V_{oc} = -2.3\frac{mV}{°C}\Delta T_c$$

1.3 A module datasheet states the following module parameters: I_{sc} = 3A; V_{oc} = 20.4V; P_{max} = 45.9W; NOCT = 43ºC. Determine the parameters (I_{sc}, V_{oc}, FF, P_{max}) of a module formed by 34 solar cells under the following operating conditions: G = 700W/m^2 ; T_a = 34ºC.

The data given has some nuances. The short circuit current, open circuit voltage and power ratings are for when the cell temperature is 25ºC and the incident radiation power is 1000W/m^2 (and AM1.5 spectrum).

$$G = 1000\frac{W}{m^2}$$
$$T_c = 25°C$$

These are ratings of the module under standard conditions. (Remember: the NOCT is not under these conditions). From the data given we can calculate the fill factor under standard conditions:

$$P = I_{sc}V_{oc}FF$$

$$\therefore FF = \frac{P}{I_{sc}V_{oc}}$$

In this case:

$$\begin{aligned}
&\therefore FF\left(T_c = 25°C, G = 1000W/m^2\right) \\
&= \frac{P\left(T_c = 25°C, G = 1000W/m^2\right)}{I_{sc}\left(T_c = 25°C, G = 1000W/m^2\right)V_{oc}\left(T_c = 25°C, G = 1000W/m^2\right)} \\
&= \frac{45.9W}{3A \times 20.4V} \\
&= 75\%
\end{aligned}$$

We will assume that the only parameter that is significantly temperature dependent is the cell/module voltage.
However, before calculating the effect of temperature, we must remember that the short circuit current and open circuit voltage also depend on the incident radiation power. We will therefore first calculate this effect. Only after will we also then correct for the temperature.
In the previous question sheet we derived the following expressions describing the dependence of the open circuit voltage and short circuit current on incident radiation/power:

$$x = \frac{P_{inc}^{'}}{P_{inc}} = \frac{G^{'}}{G}$$

$$V_{oc}^{'} = V_{oc} + \frac{k_B T}{q}\ln x$$

$$I_L \propto P_{inc}$$
$$I_{sc} = I_L$$

$$\therefore I_{sc}^{'} = xI_{sc}$$

To determine the voltage dependence we must be careful because the expression for the variation of the voltage is for each cell. One approach is to first calculate the voltage of each individual cell:

$$V_{oc}\left(T_c = 25°C, G = 1000\,W/m^2\right) = 20.4V$$

$$V_c = \frac{V}{n_c}$$

$$n_c = 34$$

$$V_{c,oc}\left(T_c = 25°C, G = 1000\,W/m^2\right) = \frac{20.4V}{34} = 0.6V$$

We can now calculate the resultant cell voltage due to a lower irradiance:

$$\therefore V_{c,oc}\left(T_c = 25°C, G = 700\,W/m^2\right) = 0.6V + \frac{1.38\times10^{-23}\,J/K \times 300K}{1.6\times10^{-19}C}\ln\left(\frac{700\,W/m^2}{1000\,W/m^2}\right)$$

$$= 0.591V$$

And the subsequent module voltage:

$$V_{oc} = n_c V_{c,oc}$$

$$= 34 \times 0.591V$$

$$= 20.1V$$

Overall, there is a decrease of 0.3V for the module.
What will vary significantly is the short circuit current:

$$I_{sc}\left(T_c = 25°C, G = 700\,W/m^2\right) = \frac{700\,W/m^2}{1000\,W/m^2}\times 3A$$

$$= 2.1A$$

Therefore, the parameters of the module under the following conditions are (We have assumed that the fill factor does not vary with irradiance):

$$T_c = 25°C$$

$$G = 700\,W/m^2$$

$$I_{sc}\left(T_c = 25°C, G = 700\,W/m^2\right) = 2.1A$$

$$V_{oc}\left(T_c = 25°C, G = 700\,W/m^2\right) = 20.1V$$

$$FF\left(T_c = 25°C, G = 700\,W/m^2\right) = FF\left(T_c = 25°C, G = 1000\,W/m^2\right) = 75\%$$

$$P\left(T_c = 25°C, G = 700\,W/m^2\right) = I_{sc}I_{sc}FF$$

$$= 2.1A \times 20.4V \times 75\% = 31.7\,\mathrm{W}$$

There parameters are true for when the cell temperature is 25ºC. However, we must now proceed to correct for the temperature dependence.

To determine the cell temperature we use the parameter NOMINAL OPERATING CELL TEMPERATURE (NOCT) and an empirical relationship relating the cell, ambient and NOCT temperatures and the incident radiation.

$$T_c = T_a + G\,\frac{NOCT - 20}{80}$$

NB: The irradiance here is in $[G] = mW/cm^2$

Remember that the NOCT is the temperature reached by the cells (when open circuited) under the following conditions (and the back of the cells is exposed to air):

$$\begin{aligned} G_{NOCT} &= 800\,\frac{W}{m^2} = 80\,\frac{mW}{cm^2} \\ T_{a,\mathrm{NOCT}} &= 20°C \\ V_{wind,NOCT} &= 1\frac{m}{s} \end{aligned}$$

In our case the cell temperature will therefore be:

$$\begin{aligned} T_c &= T_a + G\,\frac{NOCT - 20}{80} \\ T_a &= 34°C \\ G &= 700\,\frac{W}{m^2} = 70\,\frac{mW}{cm^2} \end{aligned}$$

$$\begin{aligned} T_c &= 34°C + 70\,\frac{mW}{cm^2} \times \frac{43°C - 20}{80} \\ &= 54.1°C \end{aligned}$$

Having determined the cell temperature we can now correct the open circuit voltage to obtain the parameters of the module under the following conditions

$$\begin{aligned} &T_c\left(NOCT = 43°C, T_a = 34°C, G = 700\,W/m^2\right) = 54.1°C \\ &G = 700\,W/m^2 \end{aligned}$$

(remember we are neglecting the effect on current and fill factor):

$$\begin{aligned} &I_{sc}\left(T_c = 54.1°C, G = 700\,W/m^2\right) = I_{sc}\left(T_c = 25°C, G = 700\,W/m^2\right) = 2.1A \\ &FF\left(T_c = 54.1°C, G = 700\,W/m^2\right) = FF\left(T_c = 25°C, G = 700\,W/m^2\right) = 75\% \end{aligned}$$

The temperature dependence for the open circuit voltage of a cell is:

$$\frac{dV_{oc}}{dT_c} \approx -2.3\frac{mV}{°C}$$

$$\therefore \Delta V_{c,oc} = -2.3\frac{mV}{°C}\Delta T_c$$

Our temperature difference is:

$$\Delta T_c = 54.1°C - 25°C = 29.1°C$$

And therefore the voltage change for each cell is:

$$\Delta V_{c,oc} = -2.3\frac{mV}{°C} \times 29.1°C = -67mV$$

We can now calculate the resultant open circuit voltage:

$$V_{c,oc}\left(T_c = 54.1°C, G = 700W/m^2\right) = V_{c,oc}\left(T_c = 25°C, G = 700W/m^2\right) + \Delta V_{c,oc}\left(T_c = 54.1°C\right)$$
$$V_{c,oc}\left(T_c = 25°C, G = 700W/m^2\right) = 0.591V$$

$$\therefore V_{c,oc}\left(T_c = 54.1°C, G = 700W/m^2\right) = 0.591V - 0.067V = 0.524V$$

To obtain the module voltage me multiply by the number of cells the module has:

$$V_{oc} = n_c V_{c,oc}$$
$$\therefore V_{oc}\left(T_c = 54.1°C, G = 700W/m^2\right) = 34 \times 0.524V$$
$$= 17.8V$$

Finally we can now calculate the expected output power of the module under the stipulated conditions:

$$G = 700W\,W/m^2$$
$$T_a = 34°C$$
$$NOCT = 43°C$$

$$\therefore T_c = 54.1°C$$

$$I_{sc}\left(T_c = 54.1°C, G = 700W/m^2\right) = I_{sc}\left(T_c = 25°C, G = 700W/m^2\right) = 2.1A$$
$$FF\left(T_c = 54.1°C, G = 700W/m^2\right) = FF\left(T_c = 25°C, G = 700W/m^2\right) = 75\%$$
$$V_{oc}\left(T_c = 54.1°C, G = 700W/m^2\right) = 17.8V$$

$$P = I_{sc}V_{oc}FF$$

$$P\left(T_c = 54.1°C, G = 700W/m^2\right) = 2.1A \times 17.8V \times 75\%$$
$$= 28W$$

1.4 A PV module is found to operate at 60ºC when T_a = 30ºC and G = 980W/m^2. Determine the NOCT of the module.

This is a simple case of taking the following equation:

$$T_c = T_a + G\frac{NOCT - 20}{80}$$

rearranging it:

$$\therefore NOCT = \frac{80}{G}\left(T_c - T_a\right) + 20$$

and performing the calculation.

$$T_c = 60°C$$
$$T_a = 30°C$$
$$G = 980\frac{W}{m^2} = 98\frac{mW}{cm^2}$$

$$NOCT = \frac{80}{98\frac{mW}{cm^2}}\left(60°C - 30°C\right) + 20$$
$$= 44.5°C$$

1.5 Determine the variation with ambient temperature (between -25ºC and +75ºC) of the power of a module (under standard 1000W/m^2) with 36 Si cells in series each with I_m = 5.85A and V_m = 0.5V at 25ºC and a NOCT=45ºC.

In this question we are going to assume that the temperature dependence of the maximum power point voltage V_m is equal to the temperature dependence of the open circuit voltage V_{oc}.

$$\frac{dV_{oc}}{dT_c} = \frac{dV_m}{dT_c} = -2.3\frac{mV}{°C}$$

$$\therefore \Delta V_m\left(\Delta T_c\right) = -2.3\frac{mV}{°C}\Delta T_c$$

$$\therefore V_m\left(T_c\right) = V_m\left(T_c = 25°C\right) - 2.3\frac{mV}{°C}\Delta T_c$$

We start by calculating the cell temperature using:

$$\begin{aligned} &T_c = T_a + G\frac{NOCT - 20}{80} \\ &NOCT = 45°C \\ &G = 1000\frac{W}{m^2} = 100\frac{mW}{cm^2} \end{aligned}$$

The resultant cell and changes in temperature are:

$$\begin{aligned} &T_c\left(T_a = 75°C\right) = 75°C + 100\frac{mW}{m^2}\frac{45°C - 20}{80} \\ &= 106°C \end{aligned}$$

$$\Delta T_c\left(T_a = 75°C\right) = 106°C - 25°C = 81°C$$

and

$$\begin{aligned} &T_c\left(T_a = -25°C\right) = -25°C + 100\frac{mW}{m^2}\frac{45°C - 20}{80} \\ &= 6°C \end{aligned}$$

$$\Delta T_c\left(T_a = -25°C\right) = 6°C - 25°C = -19°C$$

Remember that the change in temperature is dependent of several factors:

$$\Delta T_c\left(T_a, G, NOCT\right)$$

Assuming that only the voltage is dependent on temperature (i.e. current is unaffected)

$$I_m\left(T_c = 25°C, G = 1000W/m^2\right) = I_m\left(T_c = 106°C, T_a = 75°C, G = 1000W/m^2\right) = 5.85A$$

we can now calculate the resultant maximum power point voltages (V_m):

$$\Delta V_m\left(\Delta T_c\right) = -2.3\frac{mV}{°C}\Delta T_c$$
$$\Delta T_c = T_c - 25°C$$

$$\therefore V_m\left(T_c\right) = V_m\left(T_c = 25°C\right) - 2.3\frac{mV}{°C}\Delta T_c$$

The voltage of the individual cells at the higher and lower resultant cell temperatures are:

$$V_m\left(T_c = 106°C\right) = 500mV - 2.3\frac{mV}{°C}\times 81°C$$
$$= 313mV$$

$$V_m\left(T_c = 6°C\right) = 500mV - 2.3\frac{mV}{°C}\times\left(-6°C\right)$$
$$= 513mV$$

As such, the resultant power ratings for the individual cells are:

$$P\left(T_c\right) = I_m V\left(T_c\right)$$
$$I_m = 5.85A$$

$$\therefore P\left(T_c = 106°C, T_a = 75°C, G = 1000W/m^2\right) = 5.85\,\text{A}\times 0.313V = 1.83W$$
$$\therefore P\left(T_c = 6°C, T_a = -25°C, G = 1000W/m^2\right) = 5.85\,\text{A}\times 0.513V = 3.0W$$

Because the cell temperature is a function of the NOCT (which is an intrinsic cell/module property) ambient temperature and insolation, these are explicit in the final answer.
Finally, because our module has 36 cells in series:

$$V_{mod} = n_c V$$
$$n_c = 36$$
$$\therefore V_{mod,m} = n_c V_m$$

$$V_{mod,m}\left(T_c = 106°C, T_a = 75°C, G = 1000W/m^2\right) = 36\times 0.313V = 11.3V$$
$$V_{mod,m}\left(T_c = 6°C, T_a = -25°C, G = 1000W/m^2\right) = 36\times 0.513V = 18.5V$$

Because the cells are in series, the module current is equal to the cell current.

$$I_{mod} = I_c$$

As such the overall module power rating under the differing conditions are.

$$P_{mod}\left(T_c\right) = I_{mod}V\left(T_c\right)n_c = I_{mod}V_{mod,m}\left(T_c\right)$$
$$I_{mod} = I_m = 5.85A$$

$$P_{mod}\left(T_c = 106°C, T_a = 75°C, G = 1000W/m^2\right) = 5.85\,\text{A}\times 0.313V \times 36 = 66W$$
$$P_{mod}\left(T_c = 6°C, T_a = -25°C, G = 1000W/m^2\right) = 5.85\,\text{A}\times 0.513V \times 36 = 108W$$
$$V_{mod} = n_c V$$
$$n_c = 36$$
$$\therefore V_{mod,m} = n_c V_m$$

$$V_{mod}\left(T_c = 106°C, T_a = 75°C, G = 1000W/m^2\right) = 36\times 0.313V = 11.3V$$
$$V_{mod}\left(T_c = 6°C, T_a = -25°C, G = 1000W/m^2\right) = 36\times 0.513V = 18.5V$$

3 SIZING A GRID CONNECTED SYSTEM

Modules as those described in Table 1 are to be connected to an inverter with the specifications presented in Table 2. The modules' temperature range is -10 to 40ºC.

Table 1: Module specification

V_{oc}	30.2 V
V_m	24.0 V
I_{sc}	8.54 A
I_m	7.71 A
T coeff P	-0.485 %/ºC
T coeff V	-0.104 V/ºC

Table 2: Inverter specification

Max DC power	3200 W
Max DC voltage	600 V
MPP voltage range	268 - 480 V
DC nominal voltage	350 V
Min DC voltage	268 V
Max input DC current	12 A
Max output AC current	**15**

a) Determine the module voltage range.

Firstly, let us look at the temperature range given. This temperature range is most likely to be the ambient temperature. We are also not given the NOCT, so let us assume a typical value of 45ºC and 1000W/m^2 to estimate the highest temperature the cells can reach.

Our expression and parameters are:

$$T_c = T_a + G\frac{NOCT - 20}{80}$$
$$T_{a,max} = 40°C$$
$$NOCT = 45°C$$
$$G = 1000\frac{W}{m^2} = 100\frac{mW}{cm^2}$$

And therefore calculating the max cell temperature:

$$T_{c,max} = 40°C + 100\frac{mW}{cm^2}\frac{45°C - 20}{80}$$
$$\approx 70°C$$

(the cell and module temperature are the same)

There is no need to calculate the cell's/module's minimum temperature because the minimum temperature will occur when not irradiated and when the ambient temp is lowest.

Therefore:

$$T_{c,\min} = T_{a,min} = -10°C$$

The maximum module/cell voltage will occur when the system is at a) open circuit and on a b) cold day. The temperature dependence of the voltage is stipulated as:

$$\frac{\Delta V}{\Delta T} = -0.104 \frac{V}{°C}$$
$$\because \Delta V = -0.104 \frac{V}{°C} \Delta T$$

As such we can write that the absolute change in voltage is:

$$V_{oc}(T) = V_{oc}(25°C) + \Delta V = V_{oc}(25°C) - 0.104 \frac{V}{°C} \Delta T$$

And therefore the maximum voltage is reached when:

$$V_{max} = V_{oc}(T_{min}) = V_{oc}(25°C) - 0.104 \frac{V}{°C} \Delta T(T_{min})$$
$$= V_{oc}(25°C) - 0.104 \frac{V}{°C} (T_{min} - 25°C)$$

Having the minimum temperature and open circuit voltage at 25ºC:

$$V_{mod,oc}(25°C) = 30.2V$$
$$T_{min} = -10°C$$

the resultant maximum voltage is:

$$V_{mod,max} = V_{oc}(25°C) - 0.104 \frac{V}{°C} (T_{min} - 25°C)$$
$$= 30.2V + \left(-0.104 \frac{V}{°C}\right) \times (-10°C - 25°C)$$
$$= 33.8V$$

The minimum operating voltage, will be when at a) maximum power point (MPP) and b) hot day and c) with relatively low irradiation. Now remember that the voltage of a module is **not** strongly dependant on irradiation, so we can consider this to be negligible here.
We will assume that the V_m temperature shift is the same as the V_{oc} shift. In this case this will occur when the module is hottest.

$$T_{max} = 70°C$$

And calculate the decrease in the V_m:

$$V_{min} = V_m\left(T_{max}\right) = V_m\left(25°C\right) - 0.104\frac{V}{°C}\left(T_{max} - 25°C\right)$$

$$V_m\left(25°C\right) = 24.0\,\mathrm{V}$$

$$V_{m,min} = 24.0V + \left(-0.104V/°C\right)\times\left(70°C - 25°C\right)$$
$$= 19.3V$$

Each individual module is therefore expected to function and produce power when the voltage is between $19.3V < V < 33.8V$.

b) Determine the minimum number of modules in a string, considering a 2% drop loss in the DC cables and a 10% safety margin for the minimum inverter input voltage.

The inverter's minimum operating Vm is 268V. Adding a 10% margin, using the nominal voltage to determine the 10%:

$$V_{inv.,min} < V_{inv,DC} < V_{inv.max}$$

$$V_{inv.,min} = 268V$$
$$V_{inv.,max} = 480V$$
$$V_{inv.,nom.} = 350V$$

$$V_{inv.,nomi.,10\%} = 35V$$

Therefore one can consider a safety margin for the minimum voltage at the inverter of:

$$V_{inv.,min,safe} = V_{inv.,min} + 10\%V_{inv.,nomi.}$$
$$V_{inv.,min,safe} = 268V + 35V$$
$$= 303V$$

There is also the consideration of the loss of the potential in the DC cables. I.e., there will be a loss of 2% of the voltage produced by the modules, and as such we have to further increase the minimum voltage of the string of modules to compensate for this. The easiest way to think of the equation that allows us to calculate this is to think of taking the voltage produce by the modules and subtract the voltage loss to obtain the voltage at the inverter:

$$V_{inverter} = s_n V \times\left(1 - V_{DC,\%loss}\right)$$

And writing for the case of the minimum voltage:

$$V_{inv.,min,safe} = s_n V_{m,min} \times\left(1 - V_{DC,\%loss}\right)$$

(remember that $s_n V_{m,min}$ is the voltage produced by n modules connected in series in a string)

We now know enough parameters to solve for the string length:

$$V_{DC,\%loss} = 2\%$$
$$V_{inv.,min,safe} = 303V$$
$$V_{m,min} = 19.3V$$

$$s_n = \frac{V_{inv.,min,safe}}{V_{m,min} \times \left(1 - V_{DC,\%loss}\right)}$$
$$\therefore s_n = \frac{303V}{19.3V \times \left(1 - 2\%\right)}$$
$$= 16.01$$
$$\approx 16$$

c) Determine the maximum number of modules in a string, considering a 5% safety margin for the maximum inverter input voltage.

We have previously calculated that the max voltage obtained by a module is:

$$V_{mod,max} = V_{mod,\mathrm{OC}}\left(T_{min} = -10°C\right) = 33.8V$$

The stipulated max DC voltage of the inverter is 600V. Taking a 5% safety margin:

$$V_{inv,max,safe} = V_{inv,max}\left(1 - 5\%\right)$$
$$= 600V \times \left(1 - 5\%\right)$$
$$= 570V$$

Given that the voltage of a string is:

$$V_{string} = s_n V_{mod}$$

And this is the voltage at the inverter:

$$V_{string} = V_{inv.}$$

(Here we are not taking into account voltage losses in the cables. The reason for this is we are open circuit, and as such there is no current flow and so no resistive losses.)

We can calculate:

$$V_{inv,max,safe} = 570V$$
$$V_{mod,max} = 33.8V$$

$$V_{inv,max,safe} = s_n V_{mod,\max}$$

$$\begin{aligned} s_n &= \frac{V_{inv,max,safe}}{V_{mod,\max}} \\ &= \frac{570V}{33.8V} \\ &= 16.9 \\ &\approx 16 \end{aligned}$$

If we install 17 modules we will exceed the maximum voltage, and as such we have to round down to 16.

As you can see the range of voltage attainable by the modules is compatible with the inverter. This may not always be the case. Low voltages are not risky, as these simply the preclude from the inverter functioning (resulting in an overall power generation efficiency). However, high voltages are ones to be careful with as they can cause damage to the equipment. As such, if the case was that the string lengths required to maintain the lower voltage range was higher than those of the higher range, then the string length would be set to limit the maximum voltage. However, this should not be the case with careful consideration given to module-inverter matching.

d) Determine the number of strings by matching the current specifications (neglecting temperature effects).

The current in each string I_{string} is equal to the current of the individual modules I_{mod}. Therefore:

$$I_{string} = I_{mod}$$

The current produced by an array of modules arranged into several strings is the sum of all the strings. We can assume that each string will produce the same current, as such:

$$I = n_s I_{string}$$

Where I is the total current, n_s is the number of strings.

Given that total current is that received by the inverter, then:

$$I_{inverter,DC} = n_s I_{string}$$

The maximum current that can be produced by a module is the short circuit current. As such, the max current for each string is:

$$I_{string,max} = I_{mod,sc}$$

We are now in a position to work out what is the maximum number of strings we can have so as to not overload the current input of the inverter:

$$I_{inverter,DC,\max} = n_s I_{mod,sc}$$

$$I_{inverter,DC,\max} = 12A$$
$$I_{string,max} = I_{mod,sc} = 8.54A$$

$$\therefore n_s = \frac{I_{inverter,DC,\max}}{I_{mod,sc}} = 1.41$$
$$\approx 1$$

Here we always have to round down, as we cannot have partial strings. Having 2 strings would overload the current input of the inverter.

e) Compare the array DC power of the configuration specified in the previous questions to the max DC power of the inverter.

This possible mismatch raises an interesting problem. Under normal operating conditions we would install 1 string with 16 modules which would produce:

$$P_{DC} = n_s I_{mod,m} s_n V_{mod,m}$$
$$= 1 \times 7.71A \times 16 \times 24.0V$$
$$= 2961W$$

However, if we look at the max DC power of the inverter of 3200W we can see we are not far off.

Looking at the inverter MPP and the max DC current we can calculate the supposed max power the inverter could accept:

$$P_{inv,max,DC} = V_{inv.,mpp,max} \times I_{inv.,max,DC}$$
$$= 480V \times 12A$$
$$= 5760W$$

Which significantly surpasses the stated 3200W power. The point here is we have to think about the max power the inverter accepts and within that max power adjust the configuration of the modules to stay within the other parameters.